KAP 12/12

OPAKOVÁNÍ

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YESIT.CZ

$$l: y = -\frac{1}{3}x - 1$$

$P_y = ?$ $x = 0$

$y = -\frac{1}{3} \cdot 0 - 1$

$y = -1$ $P_y[0, -1]$

$P_x = ?$ $y = 0$

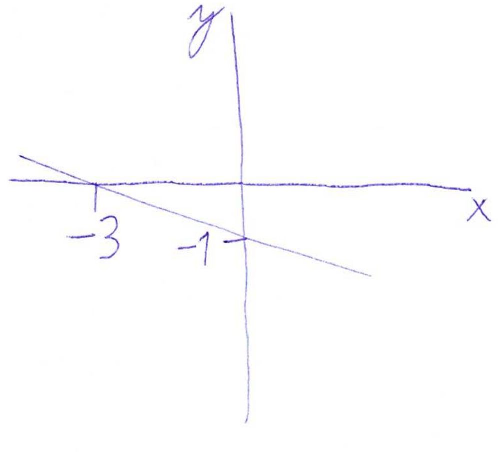
$0 = -\frac{1}{3}x - 1 \quad / + \frac{1}{3}x$

$\frac{1}{3}x = -1 \quad / : \frac{1}{3}$

$x = \dfrac{-1}{\frac{1}{3}}$

$x = -\frac{1}{1} \cdot \frac{3}{1}$

$x = -3$ $P_x[-3, 0]$



$$\boxed{\log 0{,}1 + \log (2x) = 1}$$

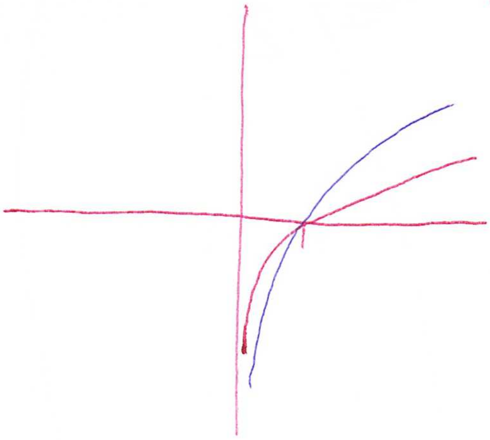
$$\log_{10} 0{,}1 + \log_{10} 2x = \log_{10} 10$$

$$\cancel{\log_{10}} 0{,}2x = \cancel{\log_{10}} 10$$

$$0{,}2x = 10 \quad | : 0{,}2$$

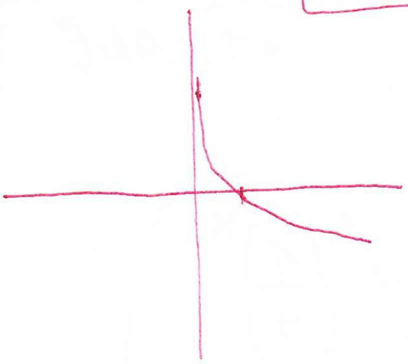
$$\underline{\underline{x = 50}}$$

$\boxed{\log x}$



$\boxed{\ln x}$

$\boxed{-\log x}$



podobně bude vypadat i

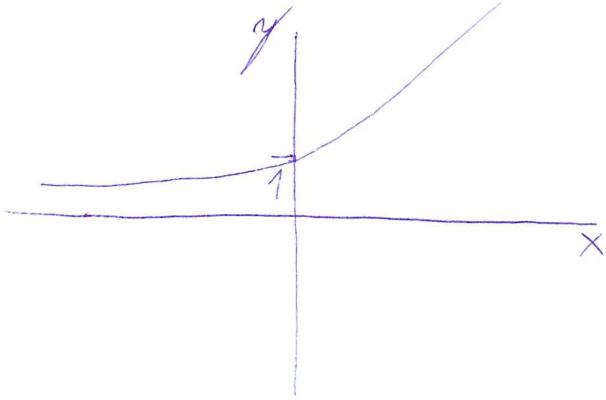
$\boxed{\log_{\frac{1}{4}} x}$

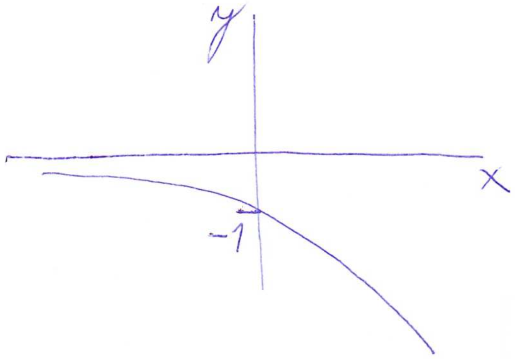
↑

je <u>menší než 1</u>

$0^0 \Rightarrow$ ERROR

$5^0 \Rightarrow 1$

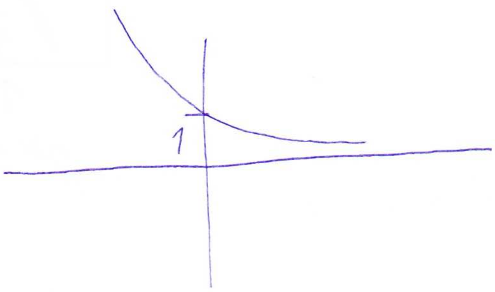


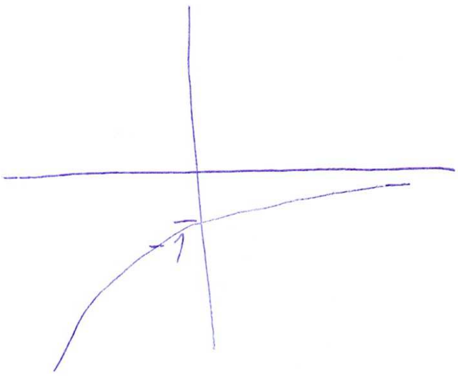


2^x

-2^x

ROSTE NA KLADNÉ STRANĚ X





$\left(\frac{1}{4}\right)^x$

$\left(-\frac{1}{4}\right)^x$

ROSTE NA MÍNUSOVÉ STRANĚ X.

ROVNICE $\boxed{3^{x+5}+7\cdot 3^{x+4}=10}$

$$3^{x+5}+7\cdot 3^{x+4}=10$$

$$3^{x}\cdot 3^{5}+7\cdot 3^{x}\cdot 3^{4}=10$$

$$3^{x}\cdot 3^{4}\cdot 3^{1}+7\cdot 3^{x}\cdot 3^{4}=10$$

$$3^{x}3^{4}(3^{1}+7)=10$$

$$3^{x}3^{4}10=10 \qquad /:10$$

$$3^{x}3^{4}=1$$

$$3^{x}3^{4}=3^{0}$$

$$\cancel{3}^{x+4}=\cancel{3}^{0}$$

$$x+4=0 \qquad /-4$$

$$\underline{\underline{x=-4}}$$

SOUSTAVA LINEÁRNÍCH NEROVNIC

Nerovnice řešíme zvlášť:

$$\boxed{\begin{aligned} -3 &\le 6x+15 \\ x+5 &> 3(x+1) \end{aligned}}$$

$-3 \le 6x + 15 \quad / -6x$

$-6x - 3 \le 15 \quad / +3$

$-6x \le 18 \quad / : (-6)$

$\underline{x \ge -3}$

$x + 5 > 3(x+1)$

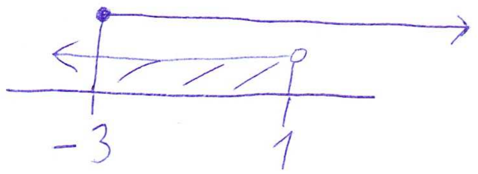
$x + 5 > 3x + 3 \quad / -3x$

$-2x + 5 > 3 \quad / -5$

$-2x > -2 \quad / \cdot (-1)$

$2x < 2 \quad / : 2$

$\underline{x < 1}$



$\underline{\underline{x \in \langle -3, 1)}}$

ZK: $x = 0$

$L_1 = -3 \quad P_1 = 15$

$L_1 < P_1$

$L_2 = 5 \quad P_2 = 3$

$L_2 > P_2$

OK :-)

V OBORU R ŘEŠTE:

$$\boxed{x(x-2)+(x-2)(x+2)=0}$$

$$x^2-2x+x^2+2x-2x-4=0$$

$$2x^2-2x-4=0 \quad /:2$$

$$x^2-x-2=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x_{1,2}=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot1\cdot(-2)}}{2\cdot1}$$

$$x_{1,2}=\frac{1\pm3}{2} \quad \begin{array}{l} \frac{4}{2}=2 \qquad x_1=2 \\ -\frac{2}{2}=-1 \qquad x_2=-1 \end{array}$$

$$K=\{2,-1\}$$

NEROVNICE: $\boxed{2x-1<-3}$

$$2x-1<-3 \quad /+1$$

$$2x<-2 \quad /:2$$

$$x<-1$$

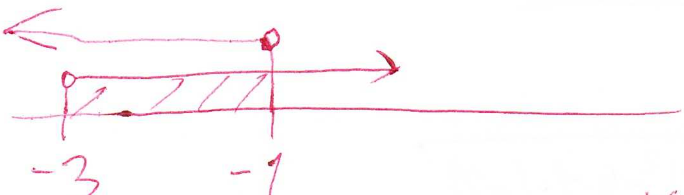
$$x\in(-\infty,1)$$

VYŘEŠ SOUSTAVU OBOU NEROVNIC

$$2x - 1 < -3$$
$$3x + 10 > 1$$

$$2x - 1 < -3 \quad /+1$$
$$2x < -2 \quad /:2$$
$$x < -1$$

$$3x + 10 > 1 \quad /-10$$
$$3x > -9 \quad /:3$$
$$x > -3$$



$$x \in (-3, -1)$$

Určete souřadnice bodu P[x,y] v němž se protínají grafy funkcí f a g:

$f: y = 2x - 9$

$g: y = 3 - 2x$

$$y = 2x - 9$$
$$y = 3 - 2x$$

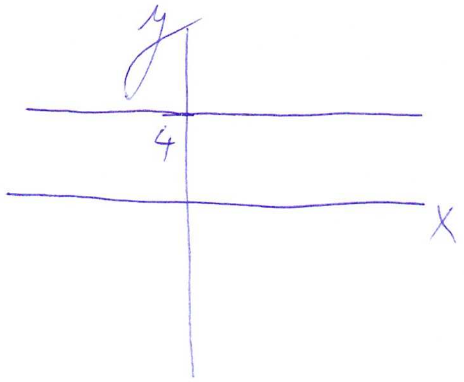
$$y - 2x = -9$$
$$y + 2x = 3 \quad /+$$

$$2y = -6 \quad /:2$$
$$\boxed{y = -3}$$

$$y = 2x - 9$$
$$-3 = 2x - 9 \quad /+9$$
$$+6 = 2x$$
$$2x = +6 \quad /:2$$
$$\boxed{x = 3}$$

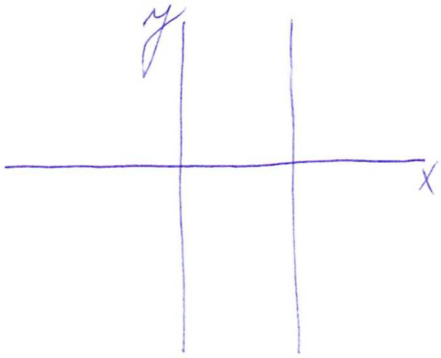
$$\underline{\underline{P[3, -3]}}$$

$f: y = 4$



KONSTANTNÍ FCE

$\not{f}:$



NENÍ FCE

$$\frac{2}{\sqrt{7}+\sqrt{2}} = \frac{2}{\sqrt{7}+\sqrt{2}} \cdot \frac{\sqrt{7}\cancel{4}-\sqrt{2}}{\sqrt{7}-\sqrt{2}} = \frac{2(\sqrt{7}-\sqrt{2})}{7-\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2}-2} =$$

$$= \underline{\underline{\frac{2(\sqrt{7}-\sqrt{2})}{5}}}$$

$$\frac{2}{\sqrt[3]{27}} = \frac{2}{\sqrt[3]{27}} \cdot \frac{\sqrt[3]{27^2}}{\sqrt[3]{27^2}} =$$

$$= \frac{2 \cdot \sqrt[3]{(3^3)^2}}{27} = \frac{2\sqrt[3]{3^6}}{27} = \frac{2 \cdot 3^{\frac{6}{3}}}{27} = \frac{2 \cdot 3^2}{27} =$$

$$= \frac{18}{27} = \frac{6}{9} = \underline{\underline{\frac{2}{3}}}$$

BINOMICKÁ VĚTA

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

K TOMU POUŽÍVÁM

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$(1+\sqrt{2})^5 = \binom{5}{0} \cdot 1^5 \cdot (\sqrt{2})^0 + \binom{5}{1} \cdot 1^4 \cdot (\sqrt{2})^1 + \binom{5}{2}\, 1^3 \cdot (\sqrt{2})^2 + \binom{5}{3} \cdot 1^2 \cdot (\sqrt{2})^3 +$$

$$+ \binom{5}{4} \cdot 1^1 \cdot (\sqrt{2})^4 + \binom{5}{5} \cdot 1^0 \cdot (\sqrt{2})^5 =$$

$$= 1 + 5\sqrt{2} + 20 + 20\sqrt{2} + 20 + 4\sqrt{2} = \underline{\underline{41 + 29\sqrt{2}}}$$

$$\binom{5}{0} = \frac{5!}{0!\,(5-0)!} = \frac{5!}{1\; 5!} = \frac{5!}{5!} = 1$$

$$\binom{5}{1} = \frac{5!}{1!\,(5-1)!} = \frac{5!}{1 \cdot 4 \cdot 3 \cdot 2 \cdot 1!} = \frac{5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1!}}{1\; \cancel{4\; 3\; 2\; 1!}} = \frac{5}{1} = 5$$

$$\binom{5}{2} = \frac{5!}{2!\,(5-2)!} = \frac{5!}{2!\;3!} = \frac{5 \cdot 4 \cdot \cancel{3!}}{2!\;\cancel{3!}} = \frac{5 \cdot 4}{2 \cdot 1!} = \frac{20}{2} = 10$$

$$\binom{5}{3} = \frac{5!}{3!\,2!} \text{ atd.}$$

$$(\sqrt{2})^5 = \left(2^{\frac{1}{2}}\right)^5 = 2^{\frac{5}{2}} = \sqrt{2^5} = \sqrt{2^2 \cdot 2^2 \cdot 2} =$$
$$= \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{2} = 4\sqrt{2}$$

$$\binom{5}{4} = \frac{5!}{4!\,(5-4)!} = \frac{5!}{4!\;1!} = \frac{5!}{4! \cdot 1} =$$

$$= \frac{5 \cdot \cancel{4!}}{1\;\cancel{4!}} = 5$$

$$\binom{5}{1} = \binom{5}{4}$$

$$\binom{5}{2} = \binom{5}{3}$$

$$(a+b)^4 = \binom{4}{0} a^4 b^0 + \binom{4}{1} a^3 b^1 + \binom{4}{2} a^2 b^2 + \binom{4}{3} a^1 b^3 + \binom{4}{4} a^0 b^4 =$$

$$= \underline{\underline{a^4 + 4a^3 b + 6a^2 b^2 + 4ab^3 + b^4}}$$

$$\binom{4}{0} = \binom{4}{4} = 1$$

$$\binom{4}{1} = \binom{4}{3} = \frac{4!}{1!\,(4-1)!} = \frac{4 \cdot \cancel{3!}}{1!\,\cancel{3!}} = \frac{4}{1} = 4$$

$$\binom{4}{2} = \frac{4!}{2!\,(4-2)!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{2!\,\cancel{2!}} = \frac{12}{2 \cdot 1} = \frac{12}{2} = 6$$

POKUD SČÍTÁM:

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

$$\binom{17}{8} + \binom{17}{9} = \underline{\underline{\binom{18}{9}}}$$

$$\binom{4}{4} + \binom{4}{3} = \underline{\underline{\binom{5}{4}}}$$

Poznámka:

$\binom{4}{4} = 1$

$\binom{5}{5} = 1$

atd.

ROVNOST

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{11}{7} + \binom{11}{5} = \binom{11}{4} + \binom{11}{5} = \underline{\underline{\binom{12}{5}}}$$

$$\binom{4}{4} + \binom{5}{4} + \binom{6}{4} + \binom{7}{4} =$$

$$= \binom{5}{5} + \binom{5}{4} + \binom{6}{4} + \binom{7}{4} =$$

$$= \binom{6}{5} + \binom{6}{4} + \binom{7}{4} =$$

$$= \binom{7}{5} + \binom{7}{4} =$$

$$= \underline{\underline{\binom{8}{5}}}$$

$$\binom{5}{0} + \binom{5}{1} + \binom{6}{2} + \binom{7}{3} + \binom{8}{4} - \binom{9}{4} =$$

$$= \binom{6}{1} + \binom{6}{2} + \binom{7}{3} + \binom{8}{4} - \binom{9}{4} =$$

$$= \binom{7}{2} + \binom{7}{3} + \binom{8}{4} - \binom{9}{4} =$$

$$= \binom{8}{3} + \binom{8}{4} - \binom{9}{4} =$$

$$= \binom{9}{4} - \binom{9}{4} = \underline{\underline{0}}$$

$$\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\binom{6}{3}+\binom{7}{3}=$$

$$=\binom{4}{4}+\binom{4}{3}+\binom{5}{3}+\binom{6}{3}+\binom{7}{3}=$$

$$=\binom{5}{4}+\binom{5}{3}+\binom{6}{3}+\binom{7}{3}=$$

$$=\binom{6}{4}+\binom{6}{3}+\binom{7}{3}=$$

$$=\binom{7}{4}+\binom{7}{3}=$$

$$=\underline{\underline{\binom{8}{4}}}$$

$$\binom{n}{k}=\frac{n!}{k!(n-k)!}$$

$$\binom{9}{3}+\binom{10}{4}+\binom{9}{2}=\binom{9}{3}+\binom{9}{2}+\binom{10}{4}=\binom{10}{3}+\binom{10}{4}=\binom{11}{4}=$$

$$=\frac{11!}{4!(11-4)!}=\frac{11\cdot 10\cdot 9\cdot 8\cdot \cancel{7!}}{4!\,\cancel{7!}}=\frac{11\cdot 10\cdot 9\cdot \cancel{8}}{\cancel{4}\cdot 3\cdot \cancel{2}\cdot 1!}=$$

$$=\frac{11\cdot 10\cdot \cancel{9}^{3}}{\cancel{3}\cdot 1}=\frac{330}{1}=\underline{\underline{330}}$$

$$\binom{n+1}{n}=\binom{n+1}{n+1-n}=\binom{n+1}{1}=\underline{\underline{n+1}}$$

$$\binom{n-3}{n-5}=\binom{n-3}{n-3-(n-5)}=\binom{n-3}{n-3-n+5}=\binom{n-3}{2}=\text{———}$$ add.

$$\frac{84}{70}=1\frac{14}{70}=$$

$$=\underline{1\frac{7}{35}}$$